Distortions in Two Sector Dynamic Models with Incomplete Specialization*

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Abstract

We extend the Jones (1971) analysis of the effects of distortions in static 2x2 trade models to the case of a two sector dynamic general equilibrium model of a small open economy with capital accumulation. In contrast to the short run results, the direction of impact of factor market distortions on steady state values do not depend on the value and physical intensity ranking of the sectors. However, the value and physical intensity rankings play an important role in the dynamics in the neighborhood of the steady state.

Differences between value and physical intensity rankings of the sectors, which gave rise to paradoxes in the static model, are shown to lead to local indeterminacy or instability in the dynamic model.

JEL Classification: F10, F11

Keywords: dynamics, trade, factor market distortions

1. Introduction

In an important paper, Jones (1971) showed how distortions in factor markets could interfere with some of the familiar comparative statics results from the two factor, two good production model that is associated with the Heckscher-Ohlin theory of international trade. The paradoxical results include the possibility that an increase in the relative price of a good results in a decrease in its supply and the possibility that a tax imposed on the earnings of a factor in one sector results in an increase in the output of that sector. One of the key findings of his paper was to show that these paradoxical outcomes arise if the

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magnitude of the distortions is such that the ranking of the sectors by the share of capital in unit costs (referred to as the "value" measure of capital intensity) differs from the ranking of sectors by capital labor ratios (the "physical" measure).[1]

The purpose of this paper is to extend the static analysis of factor market distortions to a dynamic two sector model in continuous time. This question is of interest because two sector dynamic models with constant returns to scale have arisen in a number of contexts. Examples include the literature on endogenous growth and the long run effects of factor taxation.[2] As in the static case, the existence of factor taxes can generate paradoxical results if the distortions are sufficiently large. For example, Bond, Wang and Yip (1996) show that in an endogenous growth model with accumulation of both physical and human capital the adjustment to the balanced growth path may exhibit either local indeterminacy or local instability if distortions are sufficiently large. Interestingly, the case of indeterminacy (instability) of the balanced growth path arises when the sector producing human capital is capital intensive in the physical (value) measure but labor intensive in the value (physical) measure.[3] Thus, the paradoxical behavior of the dynamic model is also linked to differences in physical and value intensity rankings.

In this paper we focus on the case of a small open economy in which one sector produces a traded consumption good and the other sector produces a non-traded investment good, so that the economy will exhibit a steady state. We show that in this case the dynamic model can be characterized by adding an intertemporal arbitrage condition and a capital accumulation equation to the full employment and zero profit conditions of the static model. This model allows us to contrast the short run and long run effects of distortions and to characterize the local dynamics in the neighborhood of the steady state for a small open economy in a particularly simple way.

This paper highlights the fact that although the potential for differences in rankings between value and physical intensities play an important role in the dynamic model, there are important differences between the static and dynamic models. One difference concerns the relationship between factor market distortions and the returns to factors of production. In the static model, the direction of the impact of factor market distortions on factor prices depends critically on whether the taxes are imposed on the capital intensive or labor-intensive sector (when measured in the value sense). In contrast, the impact on the return to capital of taxes imposed on factors in the investment good sector is the same regardless of whether the investment good sector is capital intensive or labor

[1] A second possibility is that the transformation schedule is convex to the origin. However, this possibility is not identically linked to the reversal of value and physical intensities.

[2] For example, Lucas (1988) and Bond, Wang, and Yip (1996) examine a closed economy model with physical and human capital accumulation, Bond, Trask and Wang (2003) analyse the open economy case, and Milesi-Ferreti and Roubini (1998) study the effects of factor taxation in a closed economy.

[3] Similar results are obtained by Benhabib, Meng, and Nishimura (2000) for the case where there are externalities from factor usage. They assume that the externalities are such that both the social production function and the private production functions exhibit constant returns to scale. Meng and Velasco (2004) also obtain these results in examining a model in which only capital accumulates, as in the model we consider in this paper.

intensive. The reason for this is that in the steady state, the return to capital is tied to the price of the investment good through an intertemporal arbitrage condition. A similar finding concerns the impact of factor taxes on the steady state capital stock, where all factor taxes except a tax on labor in the consumption good sector will reduce the steady state capital stock. Physical and value intensity rankings do not play a role in the comparative statics for steady state capital because of the link to the output of the investment good.

We also study the characteristics of the dynamic system in the neighborhood of the steady state. We first show that the dynamic system will have a saddle path if the rankings of the sectors on value and physical intensity are the same, although whether or not a factor price equalization property holds along the transition path depends on which sector is capital intensive. In the cases where the physical and value intensity rankings differ, the system will not have a saddle path. If the investment good sector is capital intensive in the value sense and labor-intensive in the physical sense, the system will exhibit local indeterminacy in that there will be a continuum of paths converging to the (unique) steady state. If the investment good sector is labor-intensive in the value sense and capital intensive in the physical sense, then the steady state will be unstable. The stability properties of this dynamic model contrast with those obtained by Neary (1978), who appended a dynamic adjustment process to the static model in which factors flowed in the direction of the sector in which the return was higher. His adjustment process exhibited instability in all cases where the physical and value intensity rankings differed.

We now turn to the description of the two factor, two good dynamic model of a small open economy that we analyze. We limit our attention in this paper to analysis of the interior solutions in which the economy is incompletely specialized. The potential for specialization in the case where value and physical intensities differ is explored in a companion paper (Bond and Driskill (2006)).

2. The Production Model

We consider a two good, two factor dynamic model, in which one sector produces a consumption good and the other an investment good that provides additions to the capital stock (K). The consumption good is assumed to be trade on world markets, but the investment good is non-traded. The stock of labor (L) is taken as exogenously given. Each good is produced under conditions of constant returns to scale and perfect competition using labor and capital, with factors being exchanged in competitive factor market. Factors are assumed to be imperfectly mobile between sectors of the economy, in the sense that there are exogenously given barriers to mobility that prevent the equalization of factor returns across sectors. We will refer to these barriers to mobility as sector-specific taxes, although an equivalent formulation could be given in which these barriers represent sector-specific externalities from factor usage or the existence of a fixed union wage differential. Letting w and r denote the after-tax return to labor and capital received by households and t_{ji} the tax rate on income from factor j in sector i, the pretax cost of the respective factors in sector i will be $w_i = wT_{Li}$ and $r_i = rT_{Ki}$, where $T_{ji} \equiv 1/(1 - t_{ji})$.

In the static Heckscher-Ohlin model, the production side can be solved recursively when production is incompletely specialized as has elegantly been shown by Jones (1965) for the case without distortions and Jones (1971) for the case with distortions. Assuming that both goods are produced, the competitive profit conditions in sector i require that

$$p_i = c^i\,(wT_{Li}, rT_{Ki}) \tag{1}$$

Equation (1) yields solutions for the factor prices $(\tilde{w}(p,T), \tilde{r}(p,T))$, where T denotes the vector of factor market distortions. This illustrates the factor price equalization property of the static trade model, which is that factor prices will be independent of factor supplies as long as endowments are such that both goods are produced.[4] The full employment conditions for labor and capital require that

$$\begin{aligned} c_{w_1}(wT_{L1}, rT_{K1})X_1 + c_{w_2}(wT_{L2}, rT_{K2})X_2 &= L \\ c_{r_1}(wT_{L1}, rT_{K1})X_1 + c_{r_2}(wT_{L2}, rT_{K2})X_2 &= K \end{aligned} \tag{2}$$

Given $(\tilde{w}(p,T),\tilde{r}(p,T))$ these equations yield solutions for the output levels $(\tilde{X}_1(p,K,T),\tilde{X}_2(p,K,T))$.

In the case of a static small open economy where both goods are traded, relative prices can be treated as exogenously given by world markets and equations (1) and (2) are sufficient to solve for factor prices and output levels. In the dynamic small open economy model that we consider the investment good is non-traded, so the price must be consistent with domestic market-clearing. In order for Good 2 to be held, its price must be such that the rate of return earned on the investment in the capital good is equivalent to that on alternative assets. Assuming that there is an internationally traded bond that yields a constant return equal to the discount rate (ρ) of households and that capital depreciates at rate δ, it can be shown that the household's optimization problem will require that the following intertemporal arbitrage condition hold at each point in time.[5]

$$\dot{p} = (\rho + \delta)p - r \tag{3}$$

This condition requires that the rate of return on capital net of depreciation, which is the rental rate (r/p) from a unit plus the capital gain from holding the good ($\dot{p}/p$), equal ρ.

The final equation required for the solution of the dynamic model is the equation for the accumulation of the capital good

[4] In the model with factor market distortions and two traded goods, factor price equalization across countries would hold only between countries that also had the same level of factor market distortions.

[5] Let household preferences for the consumption good be given by $U = \int_0^\infty u(C_t)e^{-\rho t}dt$. If households have access to an international bond market where they can lend or borrow at rate ρ, the household budget constraint will be $C_t = w_tL + r_tK_t + \rho B_t - \dot{B}_t - p_t(\dot{K}_t + \delta K_t)$, where B_t is the stock of bonds at time t. The existence of a traded bond with return equal to the discount rate means that the marginal utility of consumption will be equated at all points in time, so that households will have a constant level of consumption. Investment decisions in capital will be made to maximize the present value of income, so that international trade serves as a means of consumption smoothing in this model.

$$\dot{K} = X_2 - \delta K \tag{4}$$

Equations (1) to (4) characterize the production side of the dynamic open economy model. Given the solutions $\left(\tilde{w}(p,T), \tilde{r}(p,T), \tilde{X}_1(p,K,T), \tilde{X}_2(p,K,T)\right)$ for factor prices and output levels from the static model, equations (3) and (4) yield a system of differential equations in p and K. Note however that this dynamic system will also have a block recursive structure. Substituting the solution $\tilde{r}(p,T)$ into (3) will ensure that $\dot{p}$ is independent of factor supplies. We utilize this property to study the impact of factor market distortions on steady state prices in Section 3 below. In Section 4, we utilize (4) and the solution $\tilde{X}_2(p,K,T)$ from the static model to study impact of taxes on the steady state capital stock in Section 4. Finally, we study local dynamics of the system (3) and (4) in Section 5.

3. Factor Prices, Goods Prices, and Taxes

The impact of goods prices and factor taxes on factor prices with incomplete specialization can be obtained by total differentiation of (1), which yields

$$\hat{r} = \left(\frac{\theta_{L1}\hat{p} + \theta_{L1}\,\theta_{L2}(\hat{T}_{L1} - \hat{T}_{L2}) + \theta_{K1}\,\theta_{L2}\hat{T}_{K1} - \theta_{L1}\,\theta_{K2}\hat{T}_{K2}}{\theta_{K2} - \theta_{K1}} \right)$$
$$\hat{w} = -\left(\frac{\theta_{K1}\hat{p} + \theta_{K1}\,\theta_{K2}(\hat{T}_{K1} - \hat{T}_{K2}) + \theta_{K2}\,\theta_{L1}\hat{T}_{L1} - \theta_{K1}\,\theta_{L2}\hat{T}_{L2}}{\theta_{K2} - \theta_{K1}} \right) \tag{5}$$

where a "^" over a variable denotes a percentage change and $\theta_{K1} \equiv r_i c_{r_i}^{\,i}/c^i = 1 - \theta_{L1}$ is the share of capital costs in unit costs in sector i. Sector 2 is capital intensive in the value sense if $\theta_{K2} > \theta_{K1}$. These comparative statics effects are well known from Jones (1971). The first term in each expression captures the Stolper-Samuelson result: a factor will experience a more than proportional increase in its return from an increase in the price of the good in which it is used intensively and a decrease in return from an increase in the price of the other good. The remaining terms show that a factor loses from an increase in taxes on any factor in the sector in which it is used intensively, and benefits from taxes on factors in the other sector. Thus it is the sectoral in which taxes are imposed, and not the identity of the factor being taxed, that determines the impact on after tax returns in the short run at fixed output prices.

The steady state return to capital can be obtained from the intertemporal arbitrage condition (3) to be

$$\left(\frac{r}{p}\right)^{SS} = \delta + \rho \tag{6}$$

The existence of a steady state output price p^{SS} is ensured if the technologies are such that $\min_p r(p)/p \le \rho + \delta \le \max_p r(p)/p$. Since $\tilde{r}(p)/p$ is a monotone function of p for all factor intensity rankings, there can be at most one value of p^{SS}. Substituting (6) into the competitive profit condition for Good 2 yields $c^2 (w^{SS}T_{L2}, r^{SS}T_{K2}) = r^{SS}/(\rho + \delta)$. Since the unit cost function is homogeneous of degree 1 in factor prices, it follows that the steady state wage / rental ratio will be determined by the factor taxes in Sector 2 alone. Differentiating this condition yields

$$\hat{w}^{SS} - \hat{r}^{SS} = -\hat{T}_{L2} \frac{\theta_{K2}}{\theta_{L2}} \hat{T}_{K2} \tag{7}$$

The steady state wage / rental ratio will be decreasing in taxes on factors located in Sector 2. Any tax on factors in Sector 2 will raise the cost of production, which must reduce w^{SS}/r^{SS} in order to keep r^{SS}/p^{SS} constant. Combining (7) with the competitive profit condition from Sector 1 yields the solutions for the impact of taxes on the respective factor returns

$$\begin{aligned} \hat{r}^{SS} = \hat{p}^{SS} &= \theta_{L1} (\hat{T}_{L2} - \hat{T}_{L1}) + \theta_{L1}\theta_{K2} \hat{T}_{K2} / \theta_{L2} - \theta_{K1} \hat{T}_{K1} \\ \hat{w}^{SS} &= -\theta_{L1}\hat{T}_{L1} - \theta_{K1}\hat{T}_{K1} - \theta_{K1} \hat{T}_{L2} - \theta_{K1}\theta_{K2}\hat{T}_{K2} / \theta_{L2} \end{aligned} \tag{8}$$

The steady state relative price of investment goods, and hence the steady state return to capital, will be increasing in the factor taxes on the investment good sector and decreasing in the taxes on the consumption good sector. The steady state return to labor is decreasing in all of the tax rates, reflecting the shifting of taxes onto the inelastically supplied factor in the long run. A proportional increase in all taxes will raise the cost of the investment good if the investment good sector is capital intensive in the value sense. Since steady state prices are independent of factor supplies, this model will generate factor price equalization in the steady state for all countries with the same level of factor market distortions.

Equations (7) and (8) illustrate two differences important between the impact of factor taxes on factor returns between instantaneous and steady state effects. The first is that the direction of the impact of factor taxes on factor returns in the steady state does not depend on which sector is the capital intensive sector, because the return on capital is tied to the relative price of the investment good in the steady state. A second difference is that a proportional increase in all taxes will reduce the steady state wage / rental ratio, whereas in the static model there was no effect on the wage rental ratio. Increasing the tax on all factors reduces the attractiveness of investments in capital, and will thus raise the relative cost of capital services in the steady state.

4. Output Effects of Factor Taxation in the Short and Long Run

We now turn to an analysis of the impact in factor market distortions on the level of outputs and the steady state level of the capital stock. In order for the solution $(\tilde{w}(p,T), \tilde{r}(p,T))$ obtained above to be consistent with the full employment conditions (2), it must be the case that

$$\min_i \tilde{k}_i(p,T) \equiv c_{r_i}^{\,i}/c_{w_i}^{\,i} < k \equiv K/L < \max_i \tilde{k}_i(p,T) \tag{9}$$

We establish below that this condition must hold when evaluated at the steady state values of p and K if subsidies to capital are not too large. Sector i is said to be capital intensive in the physical sense if $k_i > k_j$, whereas it is capital intensive in the value sense if $r_i k_i/w_i > r_j k_j/w_j$. These two rankings will agree as long as $(T_{Ki}T_{Lj})/(T_{Kj}T_{Li}) < k_j/k_i$, which requires that the relative tax bias against capital not be too large in the physically capital intensive sector.[6]

The comparative statics for output changes with fixed factor supplies can be derived as in Jones (1971). Letting λ_{ij} denote the share of factor $j \in \{L,K\}$ employed in Sector $i \in \{1,2\}$ and $\gamma_{Li} \equiv -w_i c_{w_i w_i}^{\,i}/c_{w_i}^{\,i}$ ($\gamma_{Ki} \equiv -r_i c_{r_i r_i}^{\,i}/c_{r_i}^{\,i}$) the own price elasticity of demand for labor (capital) in Sector i,[7] we have

$$\begin{pmatrix} \lambda_{L1} & \lambda_{L2} \\ \lambda_{K1} & \lambda_{K2} \end{pmatrix} \begin{pmatrix} \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \begin{pmatrix} \hat{L} + \sum_{i=1}^{2} \lambda_{Li}\gamma_{Li}\,(\hat{w} - \hat{r} + \hat{T}_{Li} - \hat{T}_{Ki}) \\ \hat{K} - \sum_{i=1}^{2} \lambda_{Ki}\gamma_{Ki}\,(\hat{w} - \hat{r} + \hat{T}_{Li} - \hat{T}_{Ki}) \end{pmatrix} \tag{10}$$

The terms on the right hand side of (10) show that at constant factor prices, an increase in T_{Li}/T_{Ki} will increase the demand for capital and reduce the demand for labor. This can be thought of as the "factor demand" effect of factor taxation, because its impact depends on which factor is being taxed. It follows from (5) that there will also be an indirect effect of factor taxation, since changes in factor taxes on the capital (labor) intensive sector will raise (lower) w/r. This latter effect can be described as the "output tax" effect of factor taxation, since its impact depends on the sector in which the tax is imposed.

[6] Meng and Velasco (2004) analyse a case in which the sectoral production function takes the form $X_i = L_i^{\alpha_i} K_i^{\beta_i} \bar{K}_i^{1-\alpha_i-\beta_i}$, where $\alpha_i > 0$, $\beta_i > 0$, $1 - \alpha_i - \beta_i \geq 0$, and $\bar{K}_i$ denotes the industry capital stock. The distortion in this case arises from the externality from capital, which firms ignore in making decisions on the level of capital. In this case the loci of factor prices that are consistent with factor market equilibrium (i.e. the equating of private marginal products to the factor price) will have the form $w_i = p_i \alpha_i\,(r_i/\beta_i)^{(\alpha_i-1)/\alpha_i}$. These conditions yield equilibrium conditions analogous to those in (1), where the social factor shares α_i and $(1-\alpha_i)$ play the role of the value shares and determine the impact of goods price changes on factor prices. The capital labor ratios, on the other hand, are determined by the private factor shares α_i and β_i.

[7] The elasticity of substitution in Sector i will be $\sigma_i \equiv \gamma_{Li} + \gamma_{Ki}$. The symmetry of the substitution matrix implies that $\gamma_{Li} = \theta_{Ki}\sigma_i$.

Substituting from (5) into (10) and solving yields

$$\hat{X}_1 = \frac{\lambda_{K2}\hat{L} - \lambda_{L2}\hat{K}}{\lambda_{K2} - \lambda_{L2}} - \frac{\alpha_1\hat{p} + \sum_{j\in\{K,L\}} \beta_{j1}(\hat{T}_{j1} - \hat{T}_{j2})}{(\theta_{K2} - \theta_{K1})(\lambda_{K2} - \lambda_{L2})}$$

$$\hat{X}_2 = \frac{\lambda_{L1}\hat{K} - \lambda_{K1}\hat{L}}{\lambda_{K2} - \lambda_{L2}} + \frac{\alpha_2\hat{p} + \sum_{j\in\{L,K\}} \beta_{j2}(\hat{T}_{j1} - \hat{T}_{j2})}{(\theta_{K2} - \theta_{K1})(\lambda_{K2} - \lambda_{L2})} \quad (11)$$

where $\alpha_i \equiv \lambda_{Km}(\sum_{j=1}^{2} \lambda_{Lj}\gamma_{Lj}) + \lambda_{Lm}(\sum_{j=1}^{2} \lambda_{Kj}\gamma_{Kj}) > 0$ for $i,m \in \{1,2\}$, $m \neq i$ and

$\beta_{ji} \equiv \lambda_{Km}(\lambda_{L1}\gamma_{L1}\theta_{j2} + \lambda_{L2}\gamma_{L2}\theta_{j1}) + \lambda_{Lm}(\lambda_{K1}\gamma_{K1}\theta_{j2} + \lambda_{K2}\gamma_{K2}\theta_{j1}) > 0$ for $j \in \{K,L\}$; $i,m \in \{1,2\}$, $m \neq i$. The first term in each expression is the Rybczynski effect, which shows that an increase in the stock of a factor of production will increase the output of the good that uses that factor intensively (in the physical sense) and decrease the output of the other good. It is the physical factor intensities that are relevant for the Rybczynski effect with factor market distortions.

The second term in each expression shows that an increase in the relative price of Good 2 will raise the output of Good 1 and reduce the output of Good 2 if the value and physical factor intensities agree. The remaining terms in (11) show that a tax on factors in one sector will raise the output of that sector and reduce output in the other sector when the value and factor intensities agree. The "output tax" effect dominates the "factor demand" effect, because a tax on labor or capital in Sector *i* will reduce the output of Good *i* regardless of which factor is used intensively in Sector *i*. As in the case of output price changes, a counter-intuitive result is obtained when the value and factor intensity rankings differ.

From (4), the steady state capital labor ratio will be the value $k^{SS} \equiv K^{SS}/L$ that solves $E(k) = \tilde{X}_2(p^{SS},k)/L - \delta k = 0$. The function $E(k)$ will be linear in k with $E(\tilde{k}_1) < 0$, so a unique value of k^{SS} consistent with incomplete specialization will exist if it can be shown that $E(\tilde{k}_2) > 0$. Utilizing (6) and the competitive profit conditions we have $E(k^{SS}) = (\rho + \delta)T_{k2}\, k^{SS} + w^{SS}T_{L2} - \delta k^{SS}$. A sufficient condition for $E(\tilde{k}_2) > 0$ is that $T_{K2} > \delta/(\rho + \delta)$, so that a steady state with incomplete specialization will exist as long as the subsidy to capital in the investment good sector is not too large.

The effects of factor taxes on the steady state capital stock are obtained by solving the system (10) using $\hat{X}_2 = \hat{K}$ and substituting for $\hat{w} - \hat{r}$ from (7). This yields

$$\hat{K}^{SS} = \lambda_{L1}(\gamma_{L1} + \gamma_{K1})\left(\hat{T}_{L1} - \hat{T}_{K1} - \hat{T}_{L2} - \frac{\theta_{K2}}{\theta_{L2}}\hat{T}_{K2}\right) - \frac{\lambda_{L1}\lambda_{K2}\gamma_{K2} + \lambda_{K1}\lambda_{L2}\gamma_{L2}}{\theta_{L2}\,\lambda_{K1}}\hat{T}_{K2} \quad (12)$$

The first set of terms in (12) reflect substitution effects in Sector 1 from parameter changes. Factor taxes on Sector 2 raise the relative cost of capital in Sector 1 as shown in (7), leading to substitution of labor for capital. Taxes on factors in Sector 1 will not affect $(w/r)^{SS}$, but will cause substitution between factors in Sector 1. An increase in the tax on labor (capital) in Sector 1 will result in substitution away from labor (capital) leading to an increase (decrease) in the steady state capital tax. The remaining terms in (12) capture substitution effects in Sector 2. The change in the relative factor cost in Sector 2 is given by the change in $T_{L2}w^{SS}/(T_{K2}r^{SS})$. This relative factor cost is independent of T_{L2} but decreasing in T_{K2} from (7), so only the tax on capital creates substitution of capital for labor in Sector 2.

Equation (12) shows that the effect of changes in factor taxes on the capital stock (and hence the output of the investment good) does not depend on which sector is capital intensive, or on whether the value and physical intensities agree. This stands in sharp contrast to the static results in (11). Value intensities played a role in the static output effect of tax changes because they determined the direction of change in the wage/rental ratio. However, the direction of the change in the steady state wage rental ratio in response to a tax change does not depend on the value intensity of the sectors as shown in (7). The relative physical intensities of the sectors do not matter because capital must be adjusted to match the output of Sector 2 in the steady state (i.e. the determinant of the matrix on the left hand side of (10) is simply $-\lambda_{K1}$ when $\hat{X}_2 = \hat{K}$).

5. Local Dynamics

We conclude the analysis of the equilibrium with incomplete specialization by analyzing the dynamics of the system in the neighborhood of the steady state. Linearizing the system of differential equations (3) and (4) in the neighborhood of the steady state yields

$$\begin{pmatrix} d\dot{p} \\ d\dot{K} \end{pmatrix} = \begin{pmatrix} A & 0 \\ B & \Gamma \end{pmatrix} \begin{pmatrix} p - p^{SS} \\ K - K^{SS} \end{pmatrix}$$
$$A \equiv \frac{-(\rho+\delta)(1-\theta_{K2})}{\theta_{K2}-\theta_{K1}} \qquad B \equiv \frac{X_2\alpha_2}{(\theta_{K2}-\theta_{K1})(\lambda_{K2}-\lambda_{L2})} \qquad \Gamma \equiv \frac{\delta(1-\lambda_{K2})}{\lambda_{K2}-\lambda_{L2}} \tag{13}$$

The sign of the coefficient A is determined by the Stolper-Samuelson effect from (5), and will be negative if Industry 2 is capital intensive in the value sense. Similarly, the sign of Γ is determined by the Rybczynski effect and will be positive if Good 2 is capital intensive in the physical sense. Due to the block recursive structure of the system, the diagonal elements of this matrix will be the eigenvalues. The system will exhibit saddle path stability if the eigenvalues have opposite signs, and it is clear from the definitions of the terms that the eigenvalues will have opposite signs if the value and physical intensities agree. However, the dynamics of the system will differ substantially depending on which sector is capital intensive, so we consider each case in turn.[8]

[8] The coefficient B will be positive iff the physical and value intensities agree. It is this element which reflects the perverse output-price responses when physical and value intensity rankings differ, and plays a critical role in the stability analysis conducted by Neary (1978). However, in the dynamic process studied here this term has no impact on the local stability of the system.

We begin by considering the two cases in which the value and physical intensities agree. If $\theta_{K2} > \theta_{K1}$, an increase in the price of Good 2 raises the rental on capital more than proportionally by the Stolper-Samuelson theorem. This will require a reduction in the capital gain on investment goods to restore the net return on capital to ρ, which results in a stable price adjustment process for Good 2 ($A < 0$). If $\lambda_{K2} > \lambda_{L2}$, the adjustment process of capital (at fixed p) is unstable ($\Gamma > 0$) because an increase in the capital stock will raise the output of the investment good more than proportionately. The dynamic adjustment process is illustrated in Figure 1, which shows the phase diagram for this case. The $\tilde{k}_i(p)$ loci will be downward-sloping in this case, because w/r is a decreasing function of p. The $\dot{p} = 0$ locus will be horizontal at p^{SS}, with $\dot{p} < 0$ for $p > p^{SS}$. The $\dot{k} = 0$ locus will have a slope of $-\Gamma/B$, which will be negative in the neighborhood of the steady state. The price of the investment good will decline (increase) along the saddle path as capital accumulates (decumulates) toward the steady state level. The adjustment of the price is such that it stabilizes the adjustment process of capital. It can also be seen that factor price equalization would not hold along the adjustment path to the steady state between two countries with the same factor market distortions but different capital stocks, because the relative price of the investment good varies with the level of the capital stock. The absence of factor price equalization along the transition path is not surprising, since there are two non-traded factors of production and only one traded good in this model.

Figure 1: Saddle path with $\theta_{k2} > \theta_{k1}$ and $\lambda_{k2} > \lambda_{L2}$

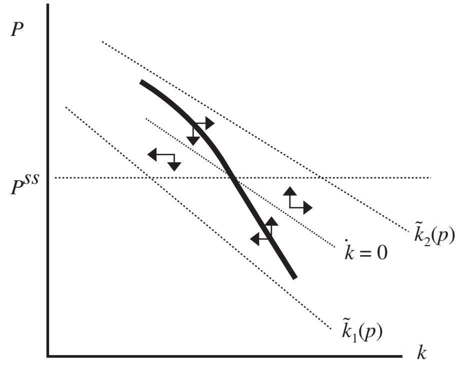
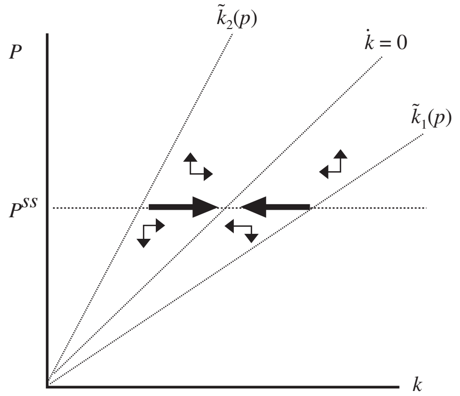


Figure 2 shows the adjustment process for the case in which $\theta_{K2} < \theta_{K1}$ and $\lambda_{K2} < \lambda_{L2}$. The slope of the $\tilde{k}_i(p)$ loci is reversed in this case, as is the slope of the $\dot{k} = 0$ locus. The price adjustment process is unstable in this case, because an increase in p reduces the return to capital and requires an increase in the rate of capital gain to satisfy the intertemporal arbitrage condition. With an unstable price adjustment process, the saddle path must be characterized by a constant value of p at the steady state level. The capital stock will converge to the steady state level with a constant price in this case because increases in the capital stock result in a decrease in the output of the investment good (i.e. $\Gamma < 0$). In this case factor price equalization would also hold along the adjustment path to the steady state for countries with identical factor market distortions.

Figure 2: Saddle path with $\theta_{k2} > \theta_{k1}$ and $\lambda_{k2} > \lambda_{L2}$



We now turn to the two cases in which physical and value intensity rankings differ. In the case where Good 2 is capital intensive in the value sense ($A < 0$) but labor intensive in the physical sense ($\Gamma < 0$), the system will exhibit local indeterminacy because the price adjustment process is stable and the capital adjustment process (at fixed prices) is also stable. Thus from an initial value of k there will be a continuum of p values for which the system will converge to the (unique) steady state. In the case where Good 2 is labor intensive in the value sense ($A > 0$) but capital intensive in the physical sense ($\Gamma > 0$), the system will exhibit dynamic instability.[9]

6. Conclusions

Our analysis has shown how factor market distortions affect the steady state price levels and stocks of capital in a small open economy with a non-traded investment good. In the long run, the fact that the return on capital is tied to the price of the investment good by the intertemporal arbitrage condition means that the relative share of capital costs in the respective sectors will not play a role in determining the impact of factor taxes on factor incomes. A similar conclusion is obtained for the steady state capital stock, where the fact that output is tied to the capital stock means that the direction of tax changes on the steady state capital stock are independent of the value and physical intensity rankings.

However, the value and physical intensities do play an important role in the dynamics of the model in the case of incomplete specialization. In particular, the model may exhibit either local indeterminacy or instability in the case where value and physical intensity rankings of sectors disagree. In the case of an unstable steady state, the analysis of the impact of taxes on steady state values becomes irrelevant as these values would not be

[9] The results on instability and indeterminacy for this case are obtained by Meng and Velasco (2004) for the case where technologies are Cobb-Douglas.

observed. In Bond and Driskill (2006), we extend this analysis to consider the possibility of equilibria with complete specialization. We show that the case where value and physical intensities disagree also leads to a multiplicity of equilibria in the static model, which generates the potential for additional dynamic paths mixing complete and incomplete specialization.

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